

Classes in HYPE

The logic HYPE has been put forward by Hannes Leitgeb [4] both as a framework to study phenomena requiring a weakening of classical logic, among which the semantic paradoxes, and as a framework to model hyperintensional phenomena. Among other things, HYPE can be seen as a framework to compare different non-classical logics, and enjoys some interesting classical recapture properties.

In a recent article by Fischer et al. [3], it has been demonstrated that HYPE performs better than other non-classical logics as a foundation for axiomatizations of Kripkean truth. However, this result depends crucially on the assumption of classical Peano Arithmetic in the background, indicating that, while HYPE is a strong non-classical logic to formulate a solution to semantic paradoxes, it does so by exploiting classical recapture results.

It is then natural to ask whether HYPE, without a classical theory in the background, and hence without appealing to recapture, can be a viable basis for non-classical foundational frameworks. To answer this, I will investigate abstraction and extensionality principles over HYPE, and compare a class theory based on HYPE to the main non-classical solutions to paradoxes of abstraction.

In particular, I will show the consistency of two abstraction principles via fixed-point models for HYPE. Further, I show that, while standard principles of extensionality are inconsistent with abstraction over HYPE, weaker axioms of extensionality are consistent with it. The class theory thus obtained, although interesting from a philosophical point of view, is shown to be proof-theoretically weak; the amount of mathematical objects and concepts which can be developed in the theory is comparable to other non-classical theories based on abstraction and extensionality, such as [2, 5].

The only significant difference with respect to other non-classical theories is a better behaviour of identity (compared to, for instance, [5]), licensed by the interaction of the axiom extensionality and the HYPE conditional. This further supports the claim that the strength of HYPE relies on classical recapture phenomena, similarly to the semantic case.

Building on this intuition, I suggest the development of a class theory extending Zermelo Fraenkel set theory over HYPE without the axiom of foundation (which can easily be shown to be equivalent to classical ZF, see [1]) with axioms for class comprehension and extensionality. This theory satisfies the desiderata of being mathematically strong while providing a non-classical treatment of the set-theoretic paradoxes.

References

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