Higher dimensional semantics of propositional theories of dependent types*

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In recent years, there has been a growing interest in various weakenings for theories of dependent types, particularly those weakenings with respect to the strength of the computation rules of the type constructors. When a dependent type theory has a type constructor that relies on a propositional equality instead of a judgemental equality, as is the case in Martin-Löf Type Theory, one says that the type constructor is in *propositional* form. Thus, a dependent type theory will have e.g. *propositional* identity types if it is endowed with a type constructor satisfying the usual rules of intensional identity types, except for the judgemental equality of its computation rule: whenever we are given judgements $x, y : A;\ p : x = y \vdash C(x, y, p) : \textsc{Type}$ and $x : A \vdash q(x) : C(x, x, r(x))$, in place of asking that the judgement $x : A \vdash J(x, x, r(x), q) \equiv q(x)$ holds -here J denotes the identity type eliminator-, we only ask that a judgement:

$$x : A \vdash H(x, q) : J(x, x, r(x), q) = q(x)$$

holds; see [2, 3] for more details.

Coquand and Danielsson [2] were the first to consider propositional identity types. This type constructor has since been extensively studied by van den Berg [3], Bocquet [4], Spadetto [6] and others. One might consider the same form of weakening for the computation rule of dependent sum types and dependent product types: these type constructors satisfying a propositional computation rule will be called *propositional dependent sum types* and *propositional dependent product types* respectively.

In this talk, we discuss a dependent type theory that includes propositional identity types, propositional dependent sum types, and propositional dependent product types, along with an arbitrary family of atomic types. We will refer to such a theory as a *propositional type theory.* The aim of this talk is to show how such a calculus admits a natural notion of semantics - generalising the one presented in [5] - via 2-dimensional categorical structures called *display map 2-categories*, where propositional type constructors are encoded as 2-dimensional categorical properties. We compare the class of models according to this semantics with the class of those derived from the usual notion of semantics for theories of dependent types, which is typically phrased via *display map categories* or *categories with attributes* (see [1]).

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