Refined Tableau Systems for Modal Logics of Confluence

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Logics of confluence, defined by instances of the axiom of confluence, cover a wide range of standard modal logics including modal logic K and axiom extensions with the axioms T, B, D, 5, alt1, Ban, G0111, G, 4 and De [5, 3, 1]. We investigate the systematic development of refined tableau systems for this class of logics and their combinations. The aim is to find an optimal set of tableau rules for each logic using methods of refinement such as decreasing branching, performing fewer inferences, and reducing application of rules which add new labels to the tableau [11].

We study tableau systems based on structural and propagation rules for these logics. Structural rules ensure that the constructed models satisfy the characteristic frame conditions of the logic [12]. Propagation rules can be seen as refinements of their structural counterparts with the idea that enough inferences are performed to ensure that a proof can be found and satisfiability can be concluded. In contrast to the structural rules, propagation rules follow the pattern of the extra axioms in such a way that whenever the antecedent of an implicational axiom is satisfied, formulae corresponding to the consequent are added to the tableaux. We study the difference in development and the properties of these rules and compare them to the rules of existing tableau calculi in the literature [2, 8, 10, 4, 9, 7, 6]. Our investigation leads to new refined tableau systems for the logics KBan, Kalt1, KG0111, KG, KDe, Kalt1De, KBG0111, KBDe and KDDe. Soundness and completeness results for all the tableau systems have been established.

Keywords: Logics of confluence · modal logics · tableau systems · refinement

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