Measuring Meaning: An Introduction to Behavioural Inferentialism.

Proof-theoretic semantics (PTS) is a major research programme in formal semantics. In this conceptual talk, I argue that there is an underdetermination problem in PTS. To address this issue, I advocate for a novel approach to PTS: *behavioural inferentialism.* This measurement-based method allows for a precise and nuanced proof-theoretic analysis of the semantic relationships between different logics.

Inferentialist approaches to semantics identify the meaning of an expression with the way we use it in inferences, inspired by Wittgenstein's 'meaning as use' [1, 2]. This inferential use, according to PTS, is determined by the expression's proof rules [3, 4, 5]. Specifically, the meaning of logical connectives is derived from their operational rules, an idea originating from Gentzen's original discussion of sequent calculi. [6, 7].

However, *exactly how* operational rules determine use and meaning has received little attention from the PTS community. The most prominent proposal is to associate the meaning of a connective with the active formulae of its operational rules [8, 9]. Yet, recent results in the logical pluralism literature indicate that this idea might be insufficient: some structural properties of sequent calculi also affect the way we use connectives [10].

To address this issue, I propose inverting the traditional order of explanation. Instead of devising proof rules 'top-down' to specify *potential* connective use, I analyse how we *actually* use a connective in proofs. In this 'bottom-up' dataset, I measure what properties of the calculus determine connective use. I argue that the minimal proof rules satisfying these properties give the use/meaning of the connective under investigation.

To implement this idea, I first add the operational rules of a connective to a *minimal* two-sided sequent calculus, only consisting of an axiom and a tool for making the two sides of the sequents interact. I argue that the rules of ID (reflexivity) and CUT (transitivity) are suitable choices for this minimal derivability relation in most interesting cases. I explain this minimality constraint as a move from Restall's conception of proof rules as *inferential patterns* [8] to Harman's *immediate inferences* [11].

Second, I prove that the connective under consideration is *definable* in this minimal calculus using Belnap's criteria of *conservativity* and *uniqueness* [12]. In this proof, I track the *inferential behaviour* of the connective, i.e. all syntactic instances and transformations of expressions containing the connective. This notion of inferential behaviour is my metric for connective use, the *independent variable.*

Third, I *restrict* one set-theoretic property of the connective rules, the *dependent variable*, and prove definability of the new rules in the same minimal calculus. Is the inferential behaviour in this proof different to the one in the previous step? If it is, the restricted property determines connective use and, thus, meaning; otherwise, it does not. Systematically repeating this process for all properties of the connective rules, one obtains a full record of all properties affecting the connective's meaning. The minimal rule satisfying these properties acts as the definition of the connective's meaning - its proof-theoretic semantic clause.

After motivating, defending and demonstrating the method of behavioural inferentialism, I discuss some key results for connectives in a range of logics, including classical, intuitionistic, dual-intuitionistic [13], minimal [14], lattice [15] and relevant logic(s) [16]. I contend that these results align well with logical practice whilst shedding new light on the relationships of connective meanings within and across different calculi. I, thus, illustrate how behavioural inferentialist semantics measures connective meaning without relying on any particular logic, akin to Davidson's 'radical interpretation' method [17].

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