Some Proof Translations for Intuitionistic Modal Logic

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The proof theory of intuitionistic and modal logics has been extensively studied in the recent decades; the advent of labelled and nested systems especially allowed for very expressive and versatile cut-free complete systems. A good way to compare different calculi for the same logic is by establishing effective translations between derivations of these. Effective translations are functions between derivations that are definable via an algorithm. This allows for direct completeness results, as well as often the transferal of structural results like rule admissibility.

As first shown by Goré and Ramanayake [2], nested sequents and tree-labelled sequents for modal logic have the same expressive power, meaning that a calculus employing one can also be expressed in terms of the other formalism. This, for example, enables one to distinguish two calculi in different formalisms.

The propositional modal logic IK has been established and developed in the 80s and 90s and was established as the intuitionistic variant of the logic K (see e.g. [1, 6]). Meanwhile, a lot of different proof systems have been developed for this logic and some of its extensions.

The only established translation between these proof systems was in [4], where Lyon compares the labelled Simpson calculus and the single-succedent nested calculus for IK together with two-sided Horn-Scott-Lemmon extensions.

In this talk, I will compare the fully labelled calculus $\text{labIK}_{\leq}$, from [5], that internalises the birelational semantics for intuitionistic modal logic, with the Maehara-style nested calculus NIKm, presented in [3]. Unlike translations that have the exact same structure embedded into them, this translation might be considered less trivial. It requires one, for instance, to add some admissible rules explicitly or to edit derivation trees before translating them. This is due $\text{labIK}_{\leq}$ being fully invertible and having strictly more structure than a simple nested sequent calculus.

This result sheds some light on how structurally different calculi behave, especially comparing a fully invertible calculus and a system with non-invertible rules. We can see that, for example, while in the Maehara-style calculus we might have to backtrack to retrieve certain formulas, we can keep all information in the full calculus. This allows one to simply analyse a formula and create a new family of variables instead of backtracking. In the future, this result might also be extended towards extensions of IK and other systems.

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