Models for Identity in Three-Valued Logics

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There is a natural way to interpret the propositional connectives and quantifiers in terms of the three semantic values 0, i, and 1, where 0 and 1 are understood as falsity and truth, and i is understood as some intermediate value [5]. These three-valued valuations do not, by themselves, determine a *logic*, because for that, you need to settle how models are used to provide a counterexample to a sequent.

If you take a counterexample to $A \succ B$ to be a model that assigns A the value 1 and B some value other than 1 (either 0 or i), the resulting logic is Kleene's strong three-valued logic, K_3 [3, see §64]. If a counterexample is a model assigning A the value 1 or i and B the value 0, the resulting logic is Priest's logic of paradox, LP [4]. If a counterexample is a model assigning A the value 1 and B the value 0, then the result is the logic ST of Strict–Tolerant validity [2]. The three logics are different generalisations of two-valued Boolean logic to a tri-valuational setting.

The logic ST is distinctive, in that it is, in some sense, a reformulation of classical logic—every classically valid sequent in this language is ST-valid [6]—but since ST allows for strictly non-classical models, there are ST theories which are not classical theories. The *Cut* rule is not unrestrictedly valid in ST. For example, if the formula P takes the value i in every model, and in the resulting theory, each sequent $A \succ P$ and $P \succ B$ is valid, while $A \succ B$ need not be valid.

There have been a number of different proposals concerning the logic of the identity predicate in this three-valued setting [1], mostly involving making minimal changes to the classical evaluation conditions, given the underlying ideology of K_3 or LP evaluations and their respective treatments of the indeterminate semantic value i. In this talk, I will use the relationship between ST evaluations and classical logic to show how there is a well-behaved class of three-valued models for the identity predicate that is *much wider* than has been previously proposed.

The key result involves characterising the three-valued models that provide no ST counterexamples to sequents valid in classical first-order predicate logic with identity. In this talk, I provide independent characterisation of such models, showing how the class generalises prior three-valued models for identity, and exploring how these models can be understood from the point of view of the logics ST, K_3 and LP.

References

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