On information theory in geometric measure theory*

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Abstract. Effective and resource-bounded dimensions were defined by Lutz in [4] and [3] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [1, 5, 2, 11] and all the references in them).
The point-to-set principle (PSP) of J. Lutz and N. Lutz [6] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [10, 8] and more recent results in [9, 13–15]).
In this talk I will review the point-to-set principles focusing on its recent extensions to separable spaces [7] and to Finite-State dimensions [12], and presenting open questions on the oracle and oracle access in PSP as well as further application opportunities.

Keywords: algorithmic dimensions · finite-state dimension · geometric measure theory · point-to-set principle.

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